MEMORANDUM REPORT M66-3-1

PERFORMANCE ANALYSIS OF DAVIS GUN LAUNCHERS FOR VEHICLES

By

SIDNEY GOLDSTEIN

AMCMS 5523.11.43400.01
DA Project No. 15531501D338

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UNITED STATES ARMY
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PHILADELPHIA, PA.

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Small Caliber Engineering Directorate

FRANKFORD ARSENAL

Philadelphia, Pa. 19137

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FOREWORD

The work described in this report was performed by the Frankford Arsenal, U. S. Army Munitions Command under AMCMS Code 5523.11.43400.01, DA Project Number 1S531501D336. Acknowledgement is made to Mr. Walter Gadomski for his contribution to the section on Gun Weight Determinations.

ABSTRACT

A set of interior ballistic equations is derived for a reactionless type of launcher of the Davis Gun type in which two masses are ejected from a common chamber.

Based on ballistic analysis of the Davis Gun launcher it appears feasible that a high performance type Davis Gun launcher could be effectively used from a vehicle.

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GLOSSARY OF SYMBOLS

A	Bore Area	in.^{2}
B	Exponential Burning Rate Coefficient	in./sec-(psi)^n
B'	Linear Burning Rate Coefficient	in./sec-psi
C	Propellant Weight	lbs
f	Fraction of the Charge Remaining at any Time t	Dimensionless
f_m	Fraction of the Charge Remaining when Pressure is a Maximum	Dimensionless
f_o	Fraction of the Charge Remaining when Shot-Start Breaks	Dimensionless
g	Acceleration Due to Gravity	ft/sec^2
ℓ	Effective Length of Initial Volume	in.
ℓ_o	True Length of Initial Volume (U_o/A)	in.
M	Weight of Projectile	lb
M'	Weight of Recoil Balancing Mass	lb
n	Burning Rate Exponent	Dimensionless
P	Instantaneous Pressure	psi
P'	Pressure at any Travel $X > X_B$	psi
P_e	Pressure at Muzzle	psi
P_B	Pressure at All-Burnt	psi
P_m	Maximum Pressure	psi

GLOSSARY (Cont'd)

P_{ss}	Pressure at which Shot-Start Breaks	psi
$\overline{T}$	Constant Temperature used to Obtain Effective Mean Impetus ($\lambda = nR\overline{T}$)	°K
T_o	Isochoric Adiabatic Flame Temperature	°K
t	Time	sec
U_o'	Initial Free Chamber Volume	$in.^3$
U_o	Total Chamber Volume	$in.^3$
U	Volume behind Projectile at any Time t	$in.^3$
V	Muzzle Velocity of Projectile	ft/sec
W	Propellant Web	in.
X	Projectile Travel at any Time	in.
X'	Travel of Recoil Balancing Mass	in.
X_B	Travel at all-Burnt	in.
X_e	Total Travel of the Projectile	in.
X_e'	Total Travel of Recoil Balancing Mass	in.
X_m	Travel when Pressure is a Maximum	in.
Y	Ratio of Final Volume to Initial Volume	Dimensionless
Y_B	Ratio of Final Volume to Volume at All-Burnt	Dimensionless
Y_B'	Ratio of the Volume Behind the Projectile at Any Travel $X > X_B$ to the Volume behind the Projectile at All-Burnt	Dimensionless

GLOSSARY (Cont'd)

β	Heat Loss Coefficient	Dimensionless
γ	Ratio of Specific Heats	Dimensionless
$\bar{\gamma}$	Effective Ratio of Specific Heats	Dimensionless
δ	Propellant Density	lb/in.^{3}
η	Co-Volume	in.^{3}/lb
λ	Effective Mean Impetus	ft-lb/lb
μ	Central Ballistic Parameter	Dimensionless
θ	Propellant Form Factor	Dimensionless
Ω	$M/M' + 1$	Dimensionless
η_k	X_b/X_e	Dimensionless
Δ_o	Loading Density	gms/cm^3

OBJECT

To obtain a generalized set of interior ballistic equations for predicting the performance of Davis Gun launchers for vehicles. Consideration is given here to cases where the projectile weight is different from that of the recoiling mass. Also, the use of a shot-start device is taken into account in the derivation of the equations.

SUMMARY

A set of interior ballistic equations is derived for a reactionless type of launcher of the Davis Gun type in which two masses are ejected from a common chamber. The important interior ballistic equations are summarized below. Travel and velocity are determined with reference to the center of mass of the system.

Velocity at all-burnt

$$V_B = \frac{AWgf_o}{2B'M}$$

Travel at all-burnt

$$X_B = \frac{\ell}{\Gamma}\left[(1 - f_o)^{\Gamma\mu\left(\frac{1-f_o}{1+\theta}\right)}(1 + \theta f_o)^{\Omega\mu\left(f_o + \frac{1-f_o}{1+\theta}\right)} - 1\right] \text{ for } \theta \neq 0$$

$$X_B = \frac{\ell}{\Gamma}\left[e^{\Omega\mu f_o}(1 - f_o)^{\Gamma\mu(1-f_o)} - 1\right] \text{ for } \theta = 0$$

Pressure at all-burnt

$$P_B = \frac{12\,C\lambda}{U_o'\left(1 - f_o\right)^{\Omega\mu\left(\frac{1-f_o}{1+\theta}\right)}\left(1 + \theta f_o\right)^{\frac{\Omega\mu}{\theta}\left(f_o - \frac{1-f_o}{1+\theta}\right)}} \quad \text{for } \theta \neq 0$$

$$P_B = \frac{12\,C\lambda}{U_o'\, e^{\Omega\mu f_o}\left(1 - f_o\right)^{\Omega\mu(1-f_o)}} \quad \text{for } \theta = 0$$

The fraction of the web remaining at peak pressure

$$f_m = \frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu} \quad \text{for all } \theta$$

The Travel at peak pressure

$$X_m = \frac{\ell}{\Omega}\left\{\left[\frac{1 - f_o}{1 - \left(\frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\Omega\mu\left(\frac{1-f_o}{1+\theta}\right)}\right.$$

$$\left.\left[\frac{1 + \theta f_o}{1 + \theta\left(\frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\frac{\Omega\mu}{\theta}\left(f_o + \frac{1-f_o}{1+\theta}\right)} - 1\right\} \quad \text{for } \theta \neq 0$$

$$X_m = \frac{\ell}{\Omega}\left\{ e\left[\frac{\Omega\mu(1 - f_o)}{\Omega\mu(1 - f_o) + 1}\right]^{\Omega\mu(1 - f_o)} - 1 \right\} \quad \text{for } \theta = 0$$

<u>**The peak pressure**</u>

$$P_m = \frac{12\,C\lambda \left[1 + \dfrac{\Omega\mu f_o(\theta - 1) + (\theta - 1)^2}{2\theta + \Omega\mu} - \theta\left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)^2\right]}{U_o'\left[\dfrac{1 - f_o}{1 - \left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\Omega\mu\left(\frac{1 - f_o}{1 + \theta}\right)}\left[\dfrac{1 + \theta f_o}{1 + \theta\left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\frac{\Omega\mu}{\theta}\left(f_o + \frac{1 - f_o}{1 + \theta}\right)}} \quad \text{for } \theta \neq 0$$

$$P_m = \frac{12C\lambda\left[\dfrac{\Omega\mu(1 - f_o) + 1}{\Omega\mu}\right]}{U_o'\; e\left[\dfrac{\Omega\mu(1 - f_o)}{\Omega\mu(1 - f_o) + 1}\right]^{\Omega\mu(1 - f_o)}} \quad \text{for } \theta = 0$$

<u>The muzzle velocity</u>

$$V_e = \sqrt{\frac{\lambda C g}{M}\left[\mu f_o^2 + \frac{2}{\Omega}\left(\frac{1 - \dfrac{1}{Y_B^{(\bar{\gamma}-1)}}}{\bar{\gamma} - 1}\right)\right]}$$

INTRODUCTION

The purpose of this report is to develop the necessary interior ballistic equations for recoilless launching systems of the Davis Gun type for vehicles.

ASSUMPTIONS

1. In the derivation of the interior ballistic equations, use was made of the isothermal model.[1] In this method, the temperature of the gases during the burning period was assumed to be a constant ($\overline{T}$) which was taken at some mean value. The effective impetus λ would then also correspond to some mean value (i.e., $\lambda = nR\overline{T}$) during the burning period.

2. The direction of stroke is assumed horizontal so that no potential energy is acquired by either body.

3. The volume (U) available to the gas behind the shot at any time is $U = U_o + \Omega AX - Cf/\delta - C(1 - f)\eta$. Assuming that $\eta = 1/\delta$, this reduces to $U = U_o + \Omega AX - C/\delta$. When $x = 0$ we define a quantity ℓ which may be interpreted as the effective length of the initial volume. $\ell = U_o - C/\delta/A = U_o'/A$. Where U_o' is the free volume behind the shot before it starts to move.

4. The rate of burning is assumed to be proportional to the pressure; $r = B'P$ (linear burning law, see Appendix A for a method of determining B' assuming the peak pressure is known).

5. There is no pressure gradient in the bore during the ballistic cycle.

6. The effective ratio of specific heats remains constant.

<u>NOTE</u>: $(\overline{\gamma} - 1) = (1 + \beta)(\gamma - 1)$

Where β = heat loss coefficient

γ = true ratio of specific heats

[1]See References 1 and 2.

7. There is no motion of the shot until the shot-start pressure (that pressure at which the shear pins break) is reached.

Thus $\ddot{X} > 0$ only when $P > P_{ss}$. The fraction of the web remaining at the time the shot-start pressure is reached is denoted by f_o.

8. The momentum and K.E. of the propellant gases is assumed to be negligible.

THEORY

Derivation of Equations

A. Basic Equations

1. Momentum Balance Equation

$$\frac{M'}{g}\frac{dV'}{dt} = \frac{M}{g}\frac{dV}{dt} \tag{1}$$

where M' is the weight of the recoiling or balancing mass and M is the weight of the projectile.

Assuming that both masses begin to move at the same time

$$M' X' = M X$$

or

$$X' = \frac{M_t}{M'} X.$$

Thus

$$X' + X = \left[\frac{M}{M'} + 1\right] X$$

The quantity

$$\frac{M}{M'} + 1$$

is denoted as Ω and thus

$$X + X' = \Omega X.$$

2. Equation of State

$$PU = 12\lambda C (1 - f)(1 + \theta \ )$$

or

$$P = \frac{12\lambda C (1 - f)(1 + \theta f)}{A(\Omega X + \ell)} \tag{2}$$

3. Equation of Motion

$$\frac{M}{g}\frac{dV}{dt} = AP \tag{3}$$

or

$$\frac{12M}{g} V \frac{dV}{dX} = AP \tag{3a}$$

4. Equation of Web Regression

$$W \frac{df}{dt} = -2B'P \tag{4}$$

B. Procedure

Eliminating P between the equation of web regression (4) and the equation of motion (3)

$$\frac{M}{g}\frac{dV}{dt} = -\frac{AW}{2B'}\frac{df}{dt}$$

With the initial condition[2] that $V = 0$ when $f = f_o$ this equation integrates to

$$V = \frac{AWg}{2B'M}(f_o - f) \tag{5}$$

or taking the derivative with respect to x :

$$\frac{dV}{dX} = -6\frac{AWg}{B'M}\frac{df}{dX} \tag{5a}$$

Starting with the equation of motion

$$\frac{12M}{g}\frac{VdV}{dX} = AP \tag{3a}$$

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[2] f_o is a constant which represents the fraction of the web remaining when the shot-start device breaks (i.e., at the time the shot begins to move). The pressure at which this occurs is denoted by P_{ss} and is determined by the material of the shot-start rod, its dimensions and the rate of loading the system. The value of f_o is obtained from the following equation (assuming P_{ss} is known)

$$P_{ss} = \frac{12C\lambda(1 - f_o)(1 + \theta f_o)}{U'_o}$$

This equation may then be solved to determine f_o

$$f_o = \frac{1}{2}\sqrt{\left(\frac{1-\theta}{\theta}\right)^2 - \left(\frac{P_{ss}U'_o - 12C\lambda}{3C\lambda\theta}\right)} - \frac{(1-\theta)}{2\theta} \quad \text{for } \theta \neq 0$$

and

$$f_o = 1 - \frac{U'_o P_{ss}}{12C\lambda} \quad \text{for } \theta = 0$$

substitute (5) for V, (5a) for dV/dX and for

$$P = \frac{12C\lambda(1 - f)(1 + \theta f)}{A(\ell + \Omega X)}$$

and solve for

$$\frac{dX}{df} = \frac{A^2 w^2 g(\Omega X + \ell)(f_o - f)}{4(B')^2 MC\lambda(1 - f)(1 + \theta f)}$$

We now define a central ballistic parameter

$$\mu = \frac{A^2 w^2 g}{4(B')^2 MC\lambda}$$

Thus

$$\frac{dX}{\Omega X + \ell} = -\mu \frac{(f_o - f)\,df}{(1 - f)(1 + \theta f)}$$

Integrating this equation with the initial conditions that at $X = 0$, $f = f_o$; we get

$$\frac{\Omega X + \ell}{\ell} = \left(\frac{1 - f_o}{1 - f}\right)^{\Omega\mu\left[\frac{1 - f_o}{1 + \theta}\right]} \left(\frac{1 + \theta f_o}{1 + \theta f}\right)^{\frac{\Omega\mu}{\theta}\left[f_o + \frac{1 - f}{1 + \theta}\right]} \quad \text{for } \theta \neq 0$$

and

$$\frac{\Omega X + \ell}{\ell} = e^{\Omega\mu(f_o - f)}\left(\frac{1 - f_o}{1 - f}\right)^{\Omega\mu(1 - f_o)} \quad \text{for } \theta = 0 \qquad (6)$$

Equation (6) gives us the travel in terms of the fraction of the web (f) remaining at any time t.

By substituting (6) into equation (2), it is now possible to determine the pressure (P) in terms of the fraction of the web (f) remains at any time t.

$$P = \frac{12C\lambda\,(1-f)(1+\theta f)}{U_o'\left(\frac{1-f_o}{1-f}\right)^{\Omega\mu\left[\frac{1-f_o}{1+\theta}\right]}\left(\frac{1+\theta f_o}{1+\theta f}\right)^{\frac{\Omega\mu}{\theta}\left[f_o-\frac{1-f_o}{1+\theta}\right]}} \quad \text{for } \theta \neq 0$$

and

$$P = \frac{12C\lambda\,(1-f)}{U_o'\,e^{\Omega\mu[f_o-f]}\left(\frac{1-f_o}{1-f}\right)^{\Omega\mu\,[1-f_o]}} \quad \text{for } \theta = 0 \tag{7}$$

It is now possible to determine the velocity, travel and pressure at all burnt (subscript B denotes the quantity when the propellant is all burnt, i.e., when $f = 0$) from equation (5) with $f = 0$.

$$V_B = \frac{AWgf_o}{2B'M} \tag{8}$$

from equation (6) with $f = 0$

$$X_B = \frac{\ell}{\Omega}\left[\left(1-f_o\right)^{\Omega\mu\left[\frac{1-f_o}{1+\theta}\right]}\left(1+f_o\right)^{\frac{\Omega\mu}{\theta}\left[f_o+\frac{1-f_o}{1+\theta}\right]} - 1\right] \quad \text{for } \theta \neq 0 \tag{9}$$

$$X_B = \frac{\ell}{\Omega}\left[e^{\Omega\mu f_o}\left(1-f_o\right)^{\Omega\mu\,[1-f_o]} - 1\right] \quad \text{for } \theta = 0$$

$$P_B = \frac{12C\lambda}{U_o'\left(1 - f_o\right)^{\Omega\mu\left[\frac{1-f_o}{1+\theta}\right]}\left(1 + \theta f_o\right)^{\frac{\Omega\mu}{\theta}\left[f_o - \frac{1-f_o}{1+\theta}\right]}} \quad \text{for } \theta \neq 0$$

And

$$P_B = \frac{12C\lambda}{U_o'\left[e^{\Omega\mu f_o}\left(1 - f_o\right)^{\Omega\mu(1-f_o)}\right]} \quad \text{for } \theta = 0 \tag{10}$$

To determine conditions at maximum pressure, the following procedure will be used. Let the subscript m denote the condition at maximum pressure.

From Equation (2) we have pressure as functions of f and x, $P = P(f, x)$, and from Equation (6) we have x as a function f, $x = x(f)$ thus

$$\frac{dP}{df} = \left(\frac{\partial P}{\partial f}\right)_x + \left(\frac{\partial P}{\partial x}\right)_f \left(\frac{dx}{df}\right) = 0 \text{ for maximum} \tag{11}$$

$$\left(\frac{\partial P}{\partial x}\right)_f = \frac{-12\,\Omega\,\lambda\,C\,(1 - f)(1 + \theta f)}{A\,(\Omega x + \ell)^2}$$

$$\left(\frac{\partial P}{\partial f}\right)_x = \frac{-12\,\lambda\,C\,(2\theta f - \theta + 1)}{A\,(\Omega x + \ell)}$$

We have already shown that

$$\frac{dx}{df} = \frac{-\mu\,(\Omega x + \ell)\,(f_o - f)}{(1 - f)(1 + \theta f)}$$

Substituting into (11) we get for f_m the fraction of the web remaining at maximum pressure

$$f_m = \frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu} \quad \text{for all } \theta \tag{12}$$

To find the travel at maximum pressure we substitute f_m for f in equation (6) and get

$$X_m = \frac{\ell}{\Omega}\left\{\left[\frac{1 - f_o}{1 - \left(\frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\Omega\mu\left(\frac{1-f_o}{1+\theta}\right)}\right.$$

$$\left.\left[\frac{1 + \theta f_o}{1 + \theta\left(\frac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\frac{\Omega\mu}{\theta}\left(f_o + \frac{1-f_o}{1+\theta}\right)} - 1\right\} \quad \text{for } \theta \neq 0$$

$$X_m = \frac{\ell}{\Omega}\left\{e\left[\frac{\Omega\mu(1 - f_o)}{\Omega\mu(1 - f_o) + 1}\right]^{\Omega\mu(1-f_o)} - 1\right\} \quad \text{for } \theta = 0 \tag{13}$$

$$P_m = \frac{12\lambda C\left[1 + \dfrac{\Omega\mu f_o(\theta - 1) + (\theta - 1)^2}{2\theta + \Omega\mu} - \theta\left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)^2\right]}{U'_o\left[\dfrac{1 - f_o}{1 - \left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\Omega\mu\left(\frac{1-f_o}{1+\theta}\right)}\left[\dfrac{1 + \theta f_o}{1 + \theta\left(\dfrac{\Omega\mu f_o + \theta - 1}{2\theta + \Omega\mu}\right)}\right]^{\frac{\Omega\mu}{\theta}\left(f_o + \frac{1-f_o}{1+\theta}\right)}} \quad \text{for } \theta \neq 0$$

And

$$P_m = \frac{12\lambda C\left[\dfrac{\Omega\mu(1 - f_o) + 1}{\Omega\mu}\right]}{U'_o\, e\left(\dfrac{\Omega\mu(1 - f_o)}{\Omega\mu(1 - f_o) + 1}\right)^{\Omega\mu(1-f_o)}} \quad \text{for } \theta = 0 \qquad (14)$$

It is impossible to have a solution with f negative at any time. Thus f_m must always be positive (i.e., $\Omega\mu f_o + \theta - 1 \geq 0$) otherwise peak pressure comes at all burnt and

$$P_m = P_B = \frac{12\lambda C}{U'_o\left[e^{\Omega\mu f_o}(1 - f_o)^{\Omega\mu(1-f_o)}\right]} \quad \text{for } \theta = 0$$

Note that in the case of a conventional gun without shot-start or recoiling parts and $\theta = 0$

$$\Omega = \lim_{W'_t \to \infty} \left(\frac{M}{M'} + 1 \right) = 1$$

$$f_o = \lim_{P_{ss} \to o} \left(1 - U'_o P_{ss} / 12 C \lambda \right) = 1$$

$$P_B = \frac{12 \lambda C}{U'_o e^{\mu}}$$

This agrees with equation (20A) P 138 of "Theory of the Interior Ballistics of Guns" by Dr. J. Corner.[3] Our solution, however, is much more general and is particularly applicable to systems using the Davis Gun principle.

To determine muzzle velocity, let subscript e denote condition at muzzle exit. K. E. of projectile at muzzle exit = K. E. of projectile at all-burnt + work done on the projectile from all burnt to muzzle exit.

$$\frac{M}{2g} V_e^2 = \frac{M}{2g} V_B^2 + \frac{1}{12} \int_{X_B}^{X_e} P' A \, dx \tag{15}$$

[3]See Reference 2.

Let Y_B' be the ratio of the volume behind the projectile at any travel $X > X_B$ to the volume behind the shot at the time of all-burnt.

$$Y_B' = \frac{\Omega X' + \ell}{\Omega X_B + \ell}$$

Using the adiabatic law for free expansion, we have the pressure (P') at any travel $X > X_B$ given by

$$P' = P_B \left(\frac{\Omega X' + \ell}{\Omega X_B + \ell} \right)^{-\bar{\gamma}} = P_B \left(Y_B' \right)^{-\bar{\gamma}}$$

Using the fact that at $X' = X_B$, $Y_B' = 1$ and $X' = X_e$, $Y_B' = Y_B$ where Y_B is the ratio of the final volume to the volume at all-burnt.

$$\frac{A}{12} \int_{X_B}^{X_e} P' dX = \frac{A P_B}{12} \left(\frac{\Omega X_B + \ell}{\Omega} \right) \int_1^{Y_B} \left(Y_B' \right)^{-\bar{\gamma}} dY_B'$$

$$= \frac{A P_B}{12} \left(\frac{\Omega X_B + \ell}{\Omega} \right) \left[\frac{1 - \dfrac{1}{Y_B^{(\bar{\gamma} - 1)}}}{\bar{\gamma} - 1} \right]$$

and remembering that

$$P_B A \left(\Omega X_B + \ell \right) = 12 \lambda C$$

thus

$$\frac{A}{12}\int_{X_B}^{X_e} P'dX = \frac{\lambda C}{\Omega} \frac{1 - \dfrac{1}{Y_B^{(\bar{\gamma}-1)}}}{\bar{\gamma} - 1} \tag{16}$$

We have already shown in equation (8) that

$$V_B = \frac{A W_o g f_o}{2B'M}$$

thus

$$V_B^2 = \frac{\mu C \lambda g}{4(B')^2 M} f_o^2 \tag{17}$$

Substituting equations (16) and (17) in equation (15) and solving for the muzzle velocity we get:

$$V_e^2 = \frac{\lambda C g}{M}\left\{\mu f_o^2 + \frac{2\left[\dfrac{1 - \dfrac{1}{Y_B^{(\bar{\gamma}-1)}}}{\bar{\gamma} - 1}\right]}{\Omega}\right\} \tag{18}$$

BALLISTIC DESIGN

For purposes of establishing some preliminary design parameters such as charge weight, total gun length, peak pressure, and also gun weight for the Davis Gun the following design ballistic quantities were selected as being typical of high performance guns.[4]

[4]See References 3 and 5.

Expansion Ratio = $\frac{\text{Final Volume}}{\text{Initial Volume}}$ (Y) = 5

$\frac{\text{Travel of projectile to all-burnt}}{\text{Total Travel}}$ (η_K) = 0.60

Loading Density (Δ_o) = 0.70 grams/cm^3

Propellant Constants See Appendix A & B

The range of projectile weights considered was from 3 to 12 lbs. and the weight of the recoiling mass was taken as 1, 2 and 3 times that of the projectile weight (i.e., Ω = 2, 3/2, 4/3). The shot-start effect was assumed negligible.

TO DETERMINE CHARGE WEIGHT

First determine Y_B, the ratio of the final volume to the volume at all-burnt

$$Y_B = \frac{\Omega X_e + \ell}{\Omega X_B + \ell}$$

since

$$Y = \frac{\Omega X_e + \ell}{\ell}$$

and

$$\eta_K = .6 = X_B / X_e$$

we have

$$\frac{Y_B}{Y} = \frac{\ell}{.6 \Omega X_e + \ell}$$

Also, since

$$\Omega X_e = \ell(Y - 1)$$

we have

$$Y_B = \frac{Y}{.6(Y - 1) + 1} = 1.47$$

for

$$Y = 5$$

With this value of Y_B, it is then possible to determine the value of

$$\left[\frac{1 - \frac{1}{(Y_B)^{\bar{\gamma}-1}}}{\bar{\gamma} - 1}\right] = .368$$

Next determine μ, the central ballistic parameter. We shall assume that no shot-start device is used so that $f_o = 1$. The travel at all-burnt is then given as

$$X_B = \frac{\ell}{\Omega}\left[e^{\Omega\mu} - 1\right]$$

and the ratio of travel to all-burnt to total travel is accordingly

$$\frac{X_B}{X_e} = \frac{\ell}{\Omega X_e}\left[e^{\Omega\mu} - 1\right] = \frac{e^{\Omega\mu-1}}{Y - 1}$$

solving for μ we have

$$\mu = \frac{1}{\Omega}\,\ell n\left[\frac{X_B(Y - 1)}{X_e} + 1\right] = \frac{1.224}{\Omega}$$

Substitution of the above values of μ and Y_B into eq. (18) yields an expression for C/M as a function of velocity and M/M'. This is plotted in Figure 1. From Figure 1 the charge weight is plotted in Figures 2, 3, and 4 for different velocities (keeping Ω fixed).

TO DETERMINE CHAMBER VOLUME (U_o)

Since we are fixing our loading density as 0.7 gms/cm^3 it is then possible by using Figures 2, 3, and 4 to plot U_o for different velocities (keeping Ω M/M' fixed). This is done in Figures 5, 6, and 7.

TO DETERMINE PEAK PRESSURES

Having previously determined M, C, and U_o it is now possible to determine peak pressure for different values of Ω. From equation (12) with $\theta = 0$,

$$f_m = 1 - \frac{1}{\Omega \mu} = 1 - \frac{1}{1.224} = .183$$

Since $f_m > 0$ we use equation (14)

$$P_m = \frac{12 \lambda C}{\Omega U_o' \mu e}$$

If $f_m < 0$ we would use equation (10)

$$P_m = P_B = \frac{12 \lambda C}{U_o' e^{\Omega \mu}}$$

For the example selected in this report it is then possible to plot P_m vs Δ_o. This is done in figure 8. At a loading density of 0.7 gms/cc our peak pressure is about 50,000 psi using eq. (14).

TO FIND TOTAL GUN LENGTH (L)

The total gun length (L) is equal to $\frac{U_o}{A}$ Y where the assumption is made that chamber area = bore area. Since chamber volume has already been calculated and plotted in Figures 5, 6, and 7 it is now possible to plot total gun length (L) for different velocities, bore diameters and projectile weights. This is done in Figures 9 through 17 inclusive.

TO DETERMINE GUN WEIGHT

In order to complete the preliminary ballistic analysis it is necessary that estimates be made of gun weight. To reduce the amount of calculations necessary, a projectile weight of 8 lbs was selected. This appears to be reasonable for the range of projectile weights considered. Before any weight calculations could be made it was first necessary to determine several ballistic quantities:

ℓ_o - True length of initial volume

X_e - Total travel of projectile

X_e' - Total travel of recoil balancing mass

P_B - Pressure at all-burnt

P_e - Muzzle Pressure

Since the total gun length L has already been calculated and plotted in Figures 9 thru 17 inclusive, it was a simple matter to determine X_e and X_e'

$$X_e + X_e' = L - \ell_o$$

$$\Omega X_e = L - \ell_o$$

$$X_e = \frac{L - \ell_o}{\Omega}$$

$$X_e' = X_e(\Omega - 1) = (L - \ell_o)(1 - 1/\Omega)$$

ℓ_o can readily be determined from Figures 5, 6, and 7 remembering that

$$\ell_o = U_o/A$$

Table I lists the values of ℓ_o, X_e, X_e' for the eight pound projectile weight with different values of Ω and gun caliber. Since at the beginning of the calculations it was assumed that $\eta_K = 0.6$, the position of the projectile and recoil balancing mass at all-burnt may be found by simply multiplying X_e and X_e' by 0.6.

To determine the pressure at all-burnt (P_B) equation (10) was used

$$P_B = \frac{12\lambda C}{U_o' e^{\Omega\mu}}$$

where

$$U_o' = U_o - C/\delta$$

Also, since

$$\Delta_o = 27.7\ C/U_o$$

where

$$\Delta_o = 0.7\ \text{gms/cm}^3$$

Table 1. VALUES OF ℓ_o, X_e AND X_e' FOR THE EIGHT-POUND PROJECTILE WEIGHT WITH DIFFERENT VALUES OF Ω AND GUN CALIBER

$$\eta_K = \frac{X_e}{X_B} = .6 \qquad M = 8 \text{ lbs}$$

V	M/M' = 1, Ω = 2			M/M' = 1/2, Ω = 3/2			M/M' = 1/3, Ω = 4/3		
(ft/sec)	ℓ_o (in.)	X_e (in.)	X_e' (in.)	ℓ_o (in.)	X_e (in.)	X_e' (in.)	ℓ_o (in.)	X_e (in.)	X_e' (in.)
				100 mm					
1000	2.42	4.84	4.84	1.82	4.85	2.43	1.63	4.85	1.62
1500	5.45	10.9	10.9	4.11	10.3	5.15	3.65	10.9	3.63
2000	9.72	19.3	19.3	7.27	19.4	9.7	6.46	19.3	6.4
2500	15.2	30.3	30.3	11.4	30.0	15.0	10.1	30.4	10.1
3000	21.9	43.7	43.7	16.4	43.6	21.8	14.6	43.7	14.6
				120 mm					
1000	1.68	3.36	3.36	1.27	3.35	1.68	1.13	3.35	1.12
1500	3.79	7.55	7.55	2.86	7.6	3.8	2.54	7.58	2.53
2000	6.75	13.6	13.6	5.05	13.5	6.75	4.49	13.4	4.5
2500	10.52	21.1	21.1	7.91	21.1	10.6	7.02	21.2	7.06
3000	15.2	30.4	30.4	11.4	30.0	15.0	10.13	30.5	10.2
				140 mm					
1000	1.23	2.46	2.46	0.93	2.45	1.23	0.83	2.45	0.82
1500	2.78	5.55	5.55	2.1	5.6	2.8	1.86	5.58	1.86
2000	4.96	9.9	9.9	3.72	9.93	4.97	3.3	9.83	3.28
2500	7.75	15.5	15.5	5.32	15.5	7.75	5.17	15.5	5.17
3000	11.2	22.3	22.3	8.37	22.2	11.1	7.45	22.4	7.46

C is the charge weight in lbs and U_o is the initial chamber volume in in.[3] we have

$$U_o' = C\left(\frac{27.7}{\Delta_o} - \frac{1}{\delta}\right)$$

and

$$P_B = \frac{12\lambda}{\left(\frac{27.7}{\Delta_o} - \frac{1}{\delta}\right)e^{\Omega\mu}} = 52,000 \text{ psi}$$

Remembering

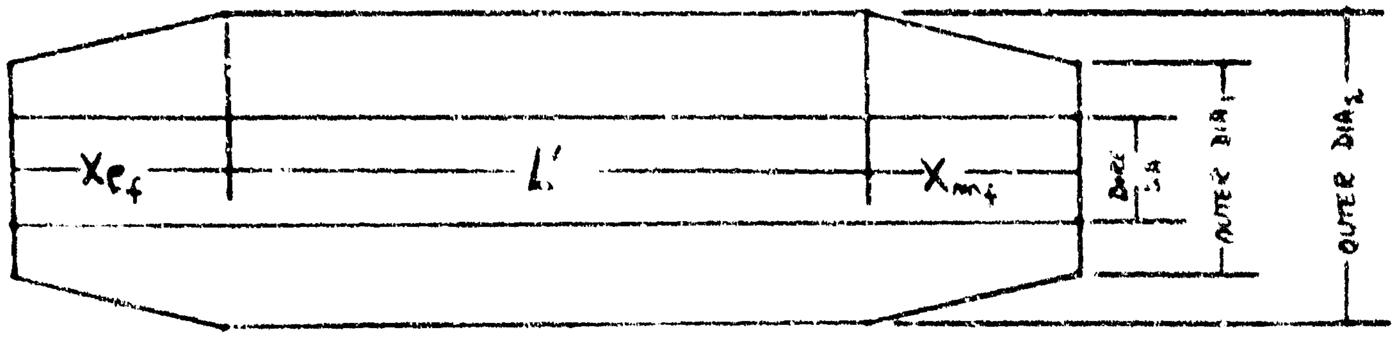
$$\frac{P_e}{P_B} = \left(Y_B\right)^{-\gamma}$$

we find that the muzzle pressure $P_e \sim 32,000$ psi

WEIGHT DETERMINATION

From Table I it was necessary to calculate an approximate gun weight for several different caliber guns using the same projectile weight. The method of calculation was the same as is shown by the following sample. It should be noted that all calculations were based on the use of an eight pound projectile, a maximum pressure of 53.000 psi and an exit or muzzle pressure of 32,000 psi.

For ease of calculation the proposed gun was assumed to have the following form:



Where

L' = length containing peak pressure

X_{ef} = length containing pressure varying from peak pressure to exit pressure (of recoil balancing mass).

X_{mf} = length containing pressure varying from peak pressure to muzzle pressure (of projectile).

At this point attention is called to the assumption made that the ratio of travel at the propellant condition "all burnt" to total travel is .6. Using this assumption it can further be assumed that the peak pressure occurs at or before the "all burnt" condition. Therefore it follows that peak pressure will occur within .6 of the travel from the chamber to either the muzzle or exit. Hence, using table I, for 100 mm at v = 2,000 ft/sec and $M/M' = 1/2$:

$$L' = .6\, X_e + .6\, X_e' + \ell_o \qquad (1)$$

$$= .6\,(19.4) + .6\,(9.7) + 7.27$$

$$= 24.73$$

$$X_{mf} = .4\, X_e \qquad (2)$$

$$= .4\,(19.4)$$

$$= 7.76$$

$$X_{ef} = .4\, X_e' \tag{3}$$

$$= .4\,(9.7)$$

$$= 3.88$$

It should be recalled that pressure (max) = 53,000 psi and pressure (muzzle or exit) = 32,000 psi. Using a yield strength of 230,000 psi for steel, the following P/YS can be calculated for the length L',

$$P/YS = \frac{\text{pressure (max)}}{\text{yield strength}} = \frac{53,000}{230,000} = .2304 \tag{4}$$

From reference (9) the wall ratio; WR is given as 1.3132. Since by definition,

$$WR = \frac{\text{Outer Diameter (O.D.)}}{\text{Inner Diameter (I.D.)}} \tag{5}$$

$$O.D._2 = I.D. \times WR \tag{6}$$

$$= 100 \times 1.3132$$

$$= 131.32 \text{ mm or } 5.170 \text{ in.}$$

Similarly, to obtain the outer diameter at the muzzle or exit, from (4)

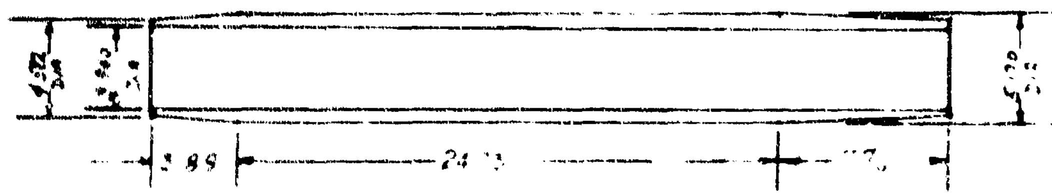
$$P/YS = \frac{32,000}{230,000} = .1391$$

$$WR = 1.1639$$

and from (6)

$$O.D._1 = 100 \times 1.1639 = 116.39 \text{ mm or } 4.582 \text{ in.}$$

Therefore the final configuration of the gun is shown in the following figure.



By using the standard equations, the volume of the gun was obtained. For example,

Volume total

$= $ Volume of cylindrical section

$+$ Volume of muzzle tapered section

$+$ Volume of exit tapered section

Volume of cylindrical section

$$= .7854\ (L')\ (\overline{OD}_2^2 - \overline{ID}^2)$$

$$= .7854\ (24.73)\ (\overline{5.170}^2 - \overline{3.940}^2)$$

$$= 217.6 \text{ in.}^3$$

Volume of muzzle tapered section

$$= .2618\ (X_{mf})\ (\overline{OD}_2^2 - (OD_2)(OD_1) + \overline{OD}_1^2) - .7854\ (X_{mf})\ \overline{ID}^2$$

$$= .2618\ (7.76)\left[(5.170)^2 + (5.170)(4.582)^2 + (4.582)^2\right]$$

$$-.7854\ (7.76)(3.940)^2$$

$$= 145.08 \quad 94.61$$

$$= 50.47 \text{ in.}^3$$

Volume of exit tapered section

$$= .2618\ (X_{ef})\ (\overline{OD}_2^2 + (OD_2)(OD_1) + \overline{OD}_1^2) - .7854\ (X_{ef})\ (\overline{ID})^2$$

$$= .2618\ (3.88) \left[(5.170)^2 + (5.170)(4.582) + (4.582)^2 \right]$$

$$= 25.24 \text{ in.}^3$$

Volume total

$$= 217.6 + 50.47 + 25.24$$

$$= 293.31 \text{ in.}^3$$

Since the density of steel is .283 lb/in.^{3}, the weight of the gun is equal to

$$.283 \times 293.31 = 83.00 \text{ lb.}$$

The density of .283 lb/in.^{3} was used in all three cases.

The results of the weight calculations for the 100 mm, 120 mm and 140 mm guns of various yield strengths using an eight pound projectile with 8, 16 and 24 pound recoil balancing masses at velocities from 1000 fps to 3000 fps are shown in Table II.

DISCUSSION

A study of Table II reveals the following:

1. The gun weight is an exponential function of velocity;

2. The weight of the gun is proportional to the muzzle energy and is dependent only on the ratio of the propelled weight to the recoil balancing weight and on the material from which the gun is fabricated;

3. For guns of the same material a saving in gun weight of approximately 22% will result when the weight of the recoil balancing weight is increased from equal to the projectile weight to twice the projectile veight;

Table II. GUN WEIGHT

Caliber (mm)	M/M'	Velocity (fps)	Y.S. = 230,000 (lb)	Y.S. = 275,000 (lb)	Y.S. = 325,000 (lb)
100	1	1000	27.6	21.3	16.8
		1500	62.5	48.2	38.0
		2000	110.8	85.6	67.4
		2500	173.7	134.2	105.8
		3000	250.6	193.6	152.8
	1/2	1000	20.8	16.1	12.7
		1500	46.9	36.2	28.5
		2000	83.0	64.2	50.5
		2500	130.3	100.6	79.4
		3000	187.9	146.5	114.4
	1/3	1000	18.6	14.3	11.3
		1500	41.7	32.2	25.4
		2000	73.9	57.1	45.0
		2500	115.7	89.4	70.5
		3000	167.1	127.6	101.8
120	1				
	1/2		SAME AS 100 mm		
	1/3				
140	1				
	1/2		SAME AS 100 mm		
	1/3				

4. For guns of the same material a saving in gun weight of approximately 33% will result when the weight of the recoil balancing weight is increased from equal to the projectile weight to three times the projectile weight;

5. For the same ratio of propelled weight to recoil balancing weight the weight of the maraged steel gun (YS = 275,000) is approximately 23% less than the high strength steel gun (YS = 230,000);

6. For the same ratio of propelled weight to recoil balancing weight the weight of the filament wound steel gun (YS = 325,000) is approximately 40% less than the high strength steel gun (YS = 230,000).

7. In the extreme case a gun made from high strength steel (YS = 230,000) and having a propelled weight to recoil balancing weight (M/M') ratio of 1/1 would be approximately 146% heavier than a filament wound steel gun (YS = 325,000) having a propelled weight to expelled weight ratio of 1/3.

8. In the middle case a gun made from high strength steel (YS = 230,000) and having a propelled weight to recoil balancing weight (M/M') ratio of 1/1 would be approximately 95% heavier than a filament wound steel gun (YS = 325,000) having a propelled weight to expelled weight of 1/3.

CONCLUSIONS

1. Based on the ballistic analysis of the Davis Gun launcher it appears feasible that a high performance type Davis Gun launcher could be effectively used from a vehicle.

2. Using recoil balancing masses which are greater than that of the projectile results in reduced gun lengths and weights. However, the decrease in gun weight must be compared with the increase in total round weight ($C + M + M'$) for any particular system.

A study of the data contained in table II for particular Davis Gun leads to the following conclusions:

1. A decrease in the ratio of propelled weight to recoil balancing weight from 1/1 to 1/3 may reduce the weight of the gun by 33% with no sacrifice of velocity or muzzle energy;

2. An increase in the strength of the material from which the gun is made, namely, from 230,000 to 325,000, may reduce the weight of the gun by 40% with no sacrifice of velocity or muzzle energy;

3. A substantial saving in the weight of the gun can be achieved by decreasing the ratio of the propelled weight to recoil balancing weight while at the same time increasing the strength of the material from which the gun is made. For example, taking an extreme case, a saving in weight of 60% can be achieved by using a filament wound steel gun with a yield strength of 325,000 and a propelled mass to expelled mass ratio of 1/3 in lieu of a high strength steel gun with a yield strength of 230,000 and a propelled mass to expelled mass ratio of 1/1.

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APPENDIX A

A METHOD OF SELECTING BURNING RATE COEFFICIENT $\left(B'\right)$

B' may be chosen such that the area under the assumed linear burning rate curve equals that under the exponential burning rate curve.

$$\int_0^{P_{max.}} B' \, P \, d\,P = \int_0^{P_{max.}} B \, P^n \, d\,P$$

$$B' = \frac{2\,B}{\left(n + 1\right) P_{max}^{(1-n)}}$$

APPENDIX B

PROPELLANT CONSTANTS USED FOR THE EXAMPLE IN THIS REPORT

λ	Impetus	3.4×10^5 ft-lbs/lb
$\bar{\gamma}$	Pseudo Ratio of Specific Heat	1.25
η	Co-Volume	28 in.^{3}/lb
δ	Propellant Density	0.06 lbs/in.^{3}
θ	Form Factor	0
B	Burning rate coefficient	4.53×10^{-3} in./sec-psi^n
n	Burning rate exponent	.7

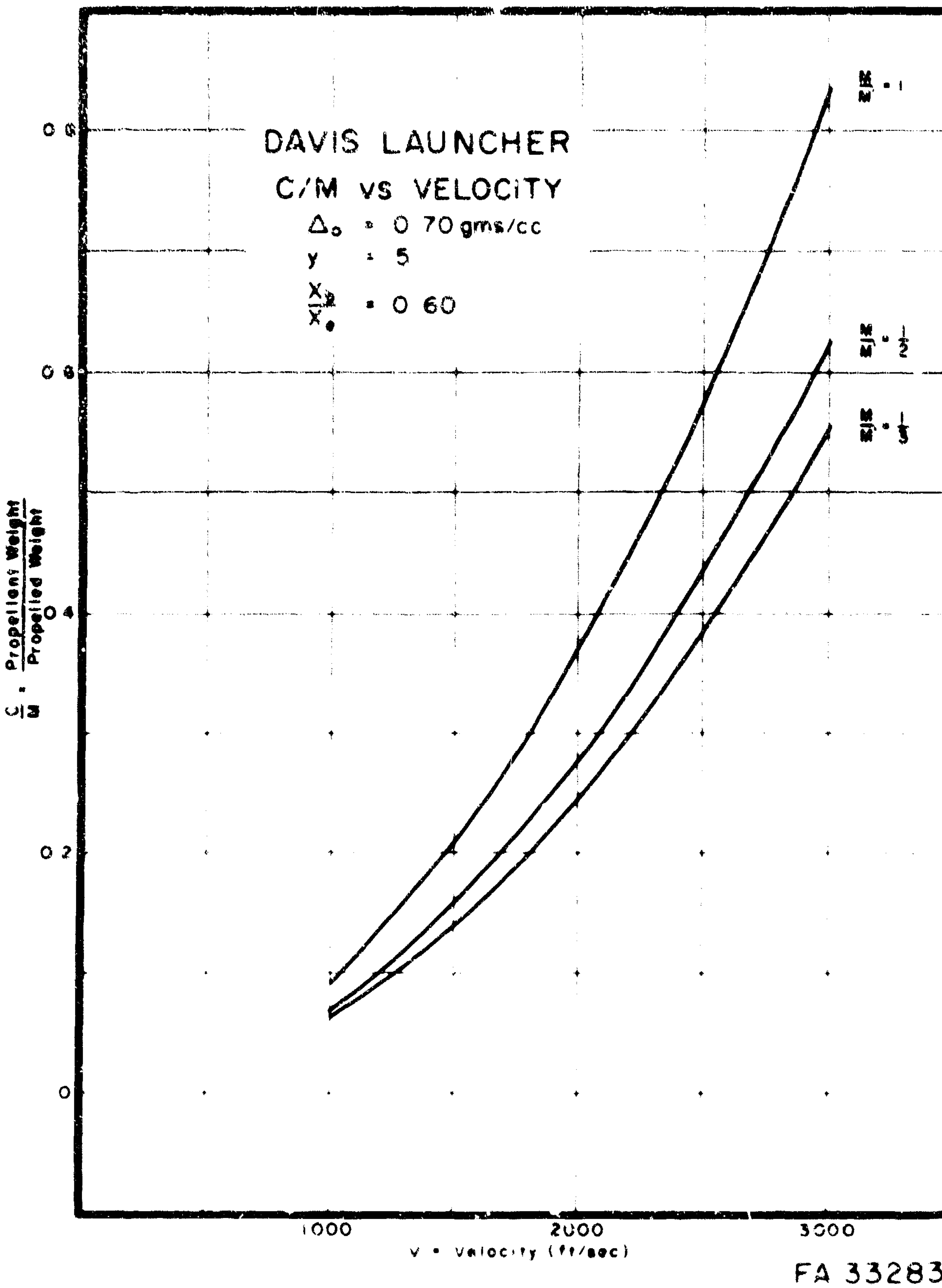

Figure 1. C/M vs Velocity

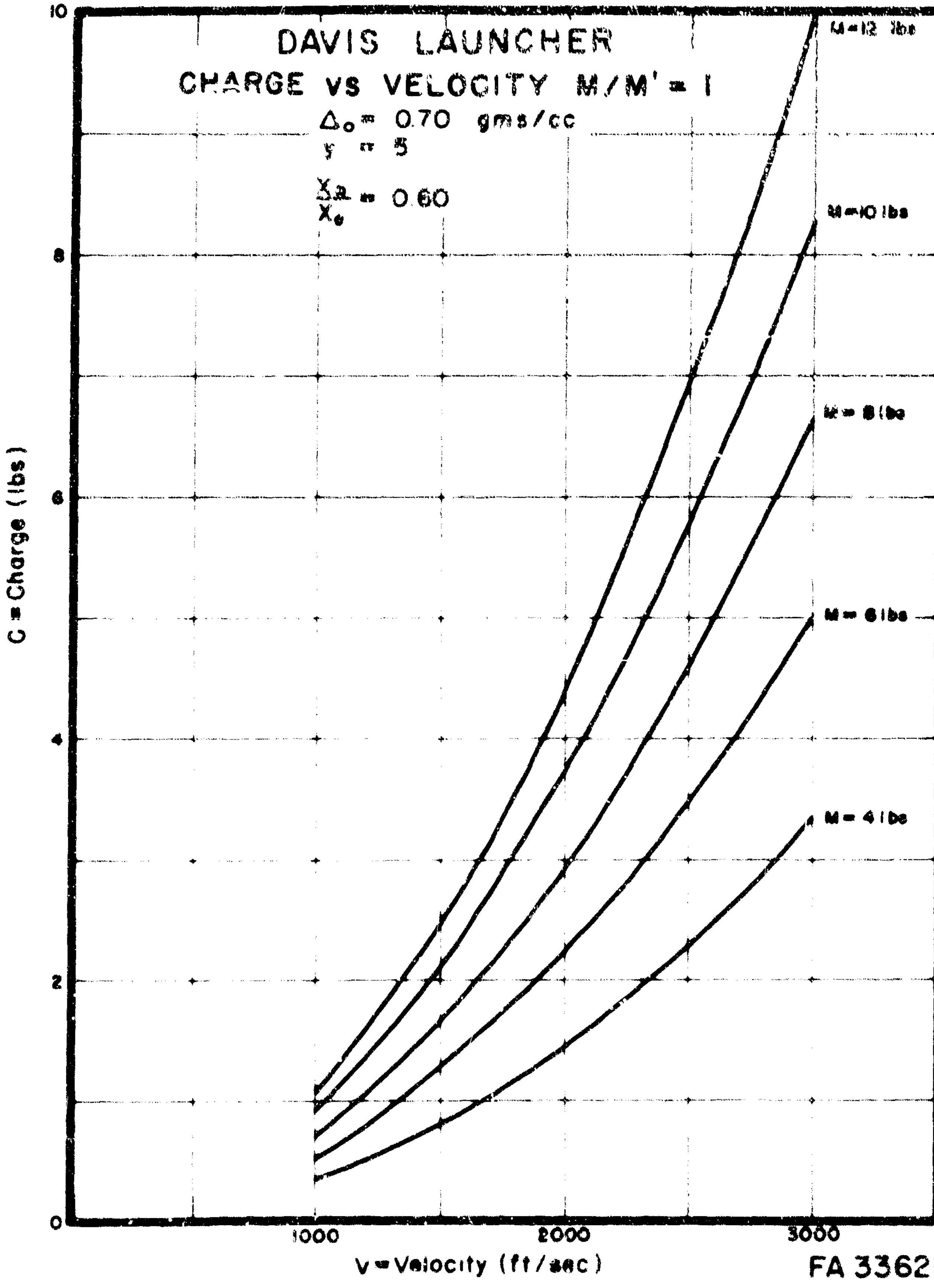

Figure 2. Charge vs Velocity M/M' = 1

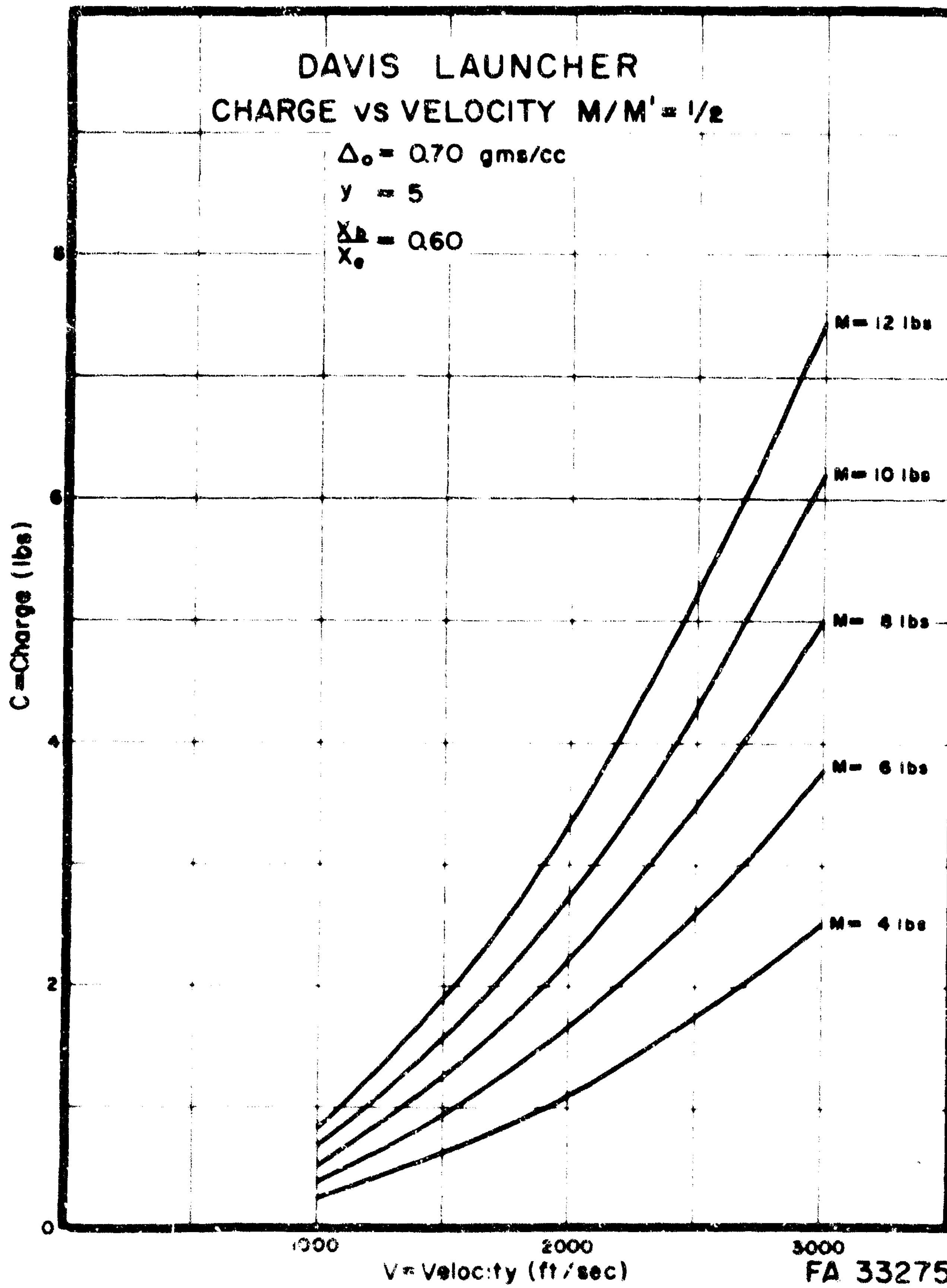

Figure 3. Charge vs Velocity $M/M' = 1/2$

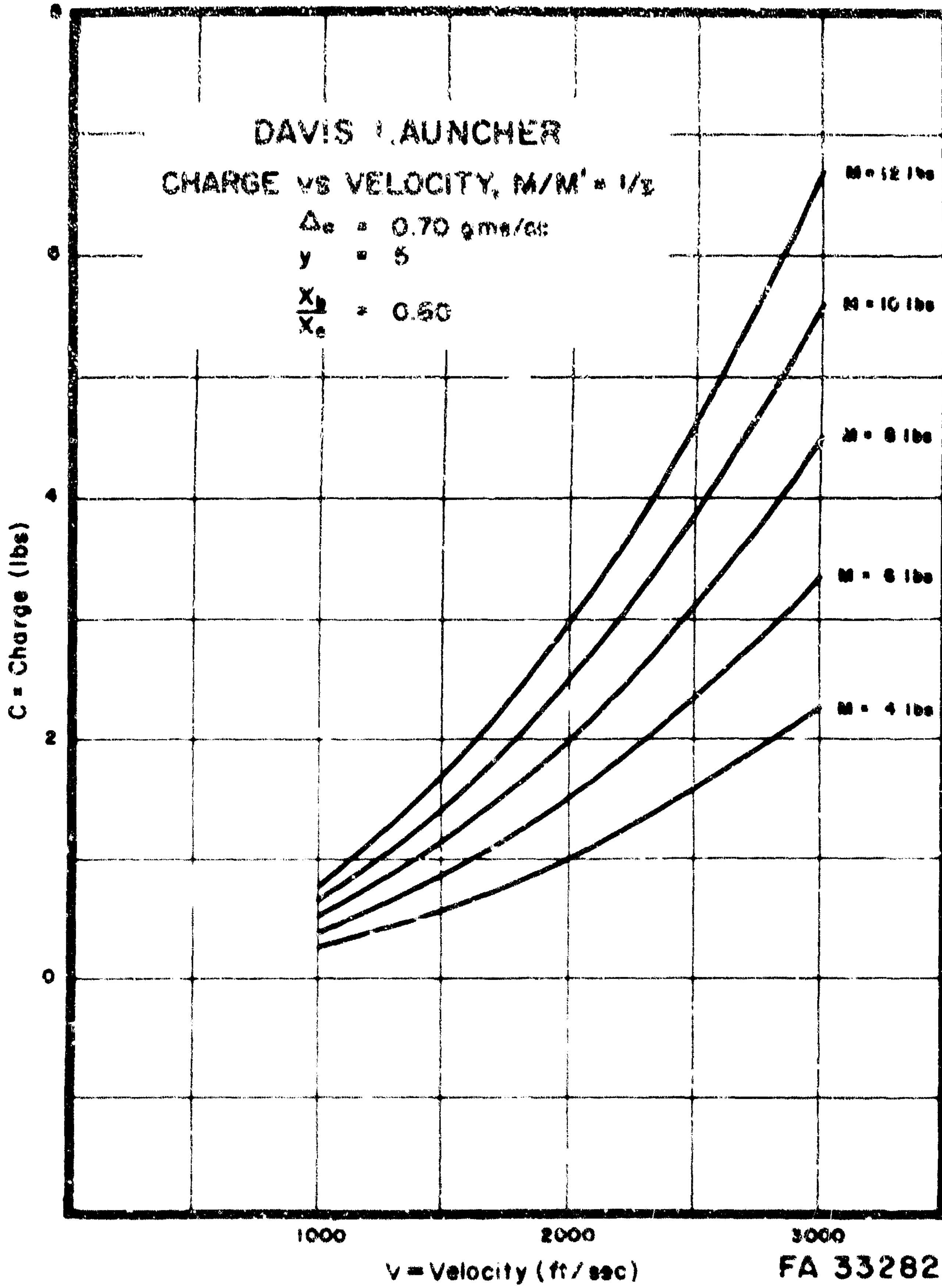

Figure 4. Charge vs Velocity M/M' = 1/3

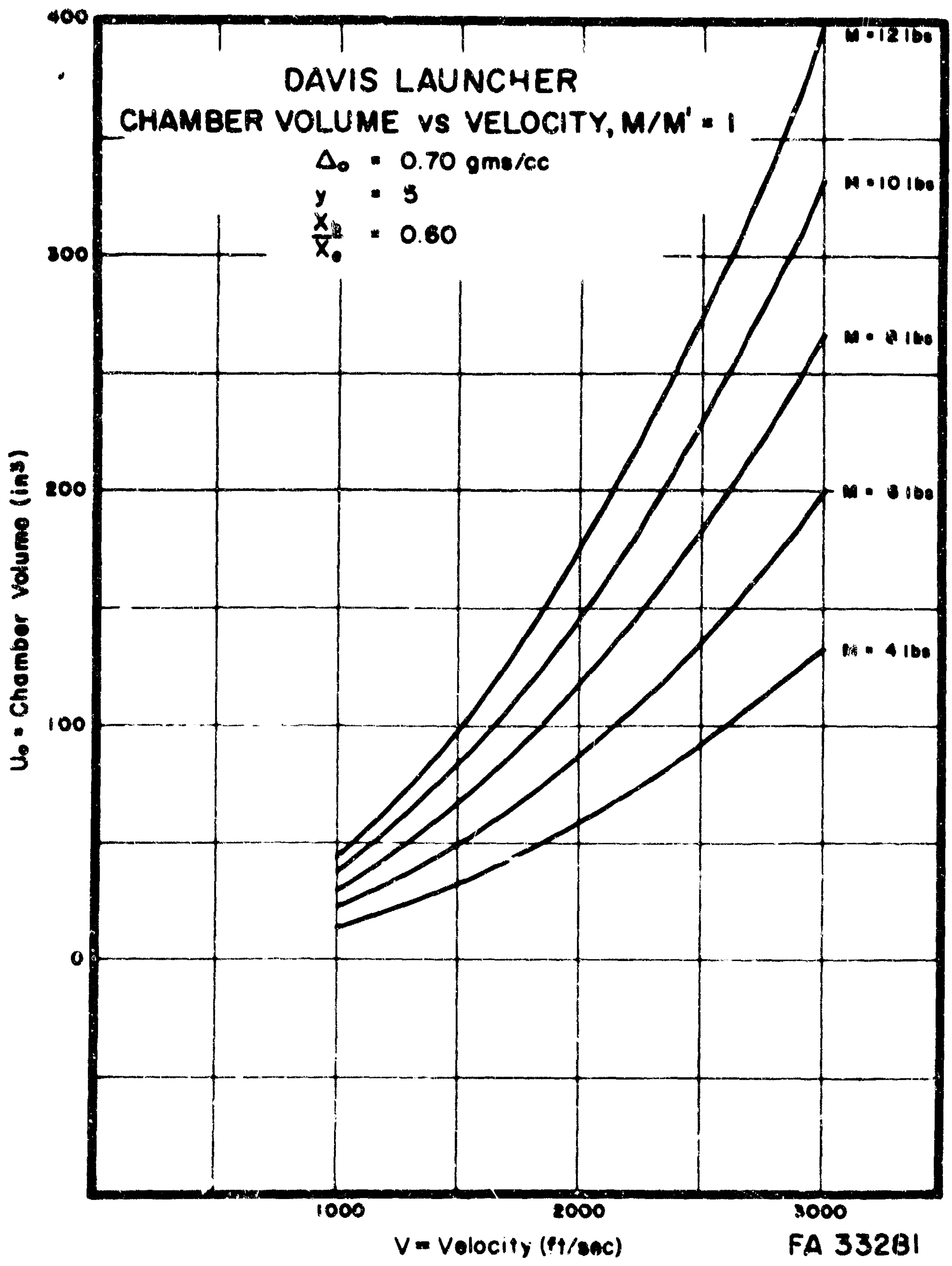

Figure 5. Chamber Volume vs Velocity $M/M' = 1$

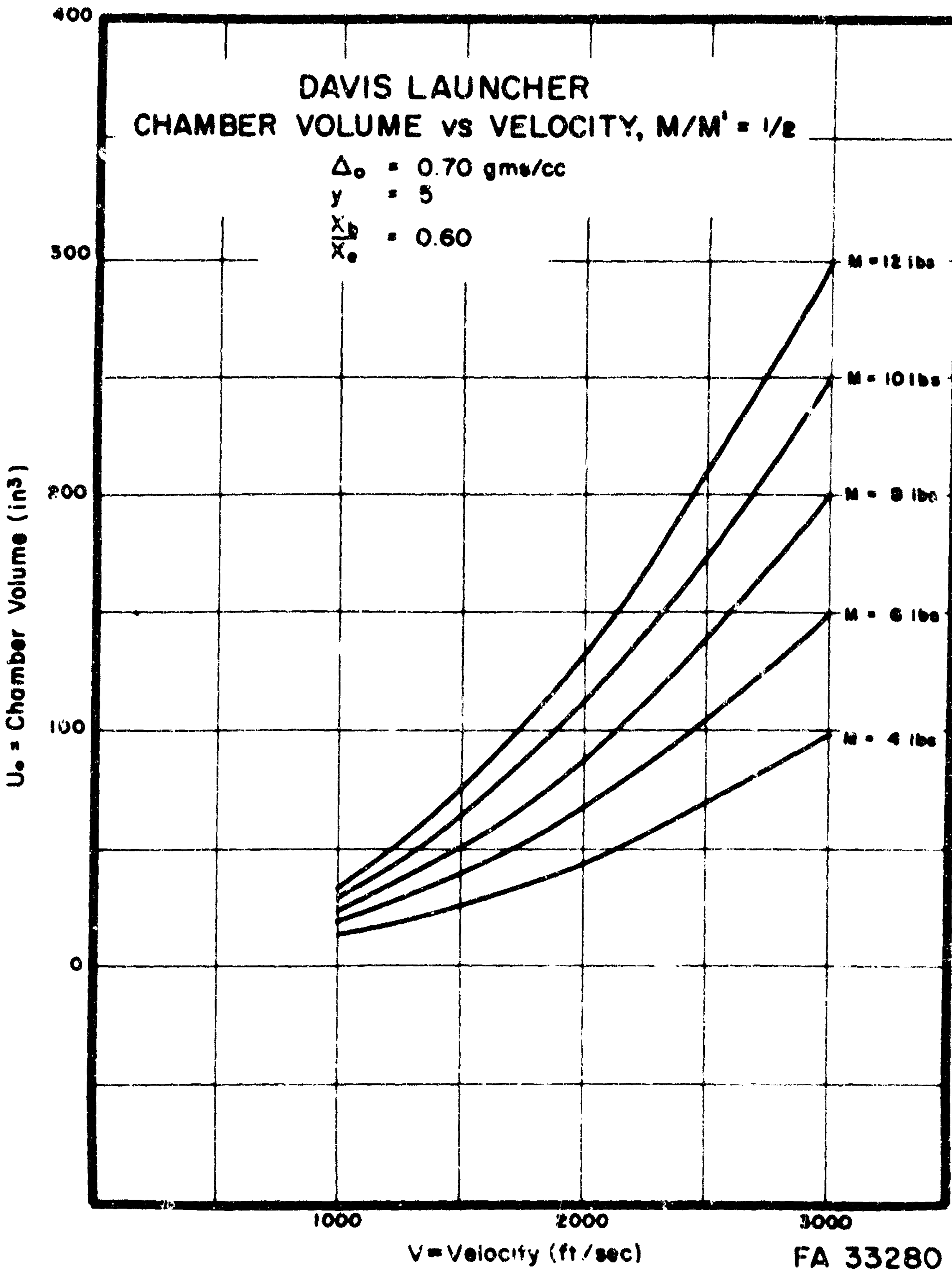

Figure 6. Chamber Volume vs Velocity $M/M' = 1/2$

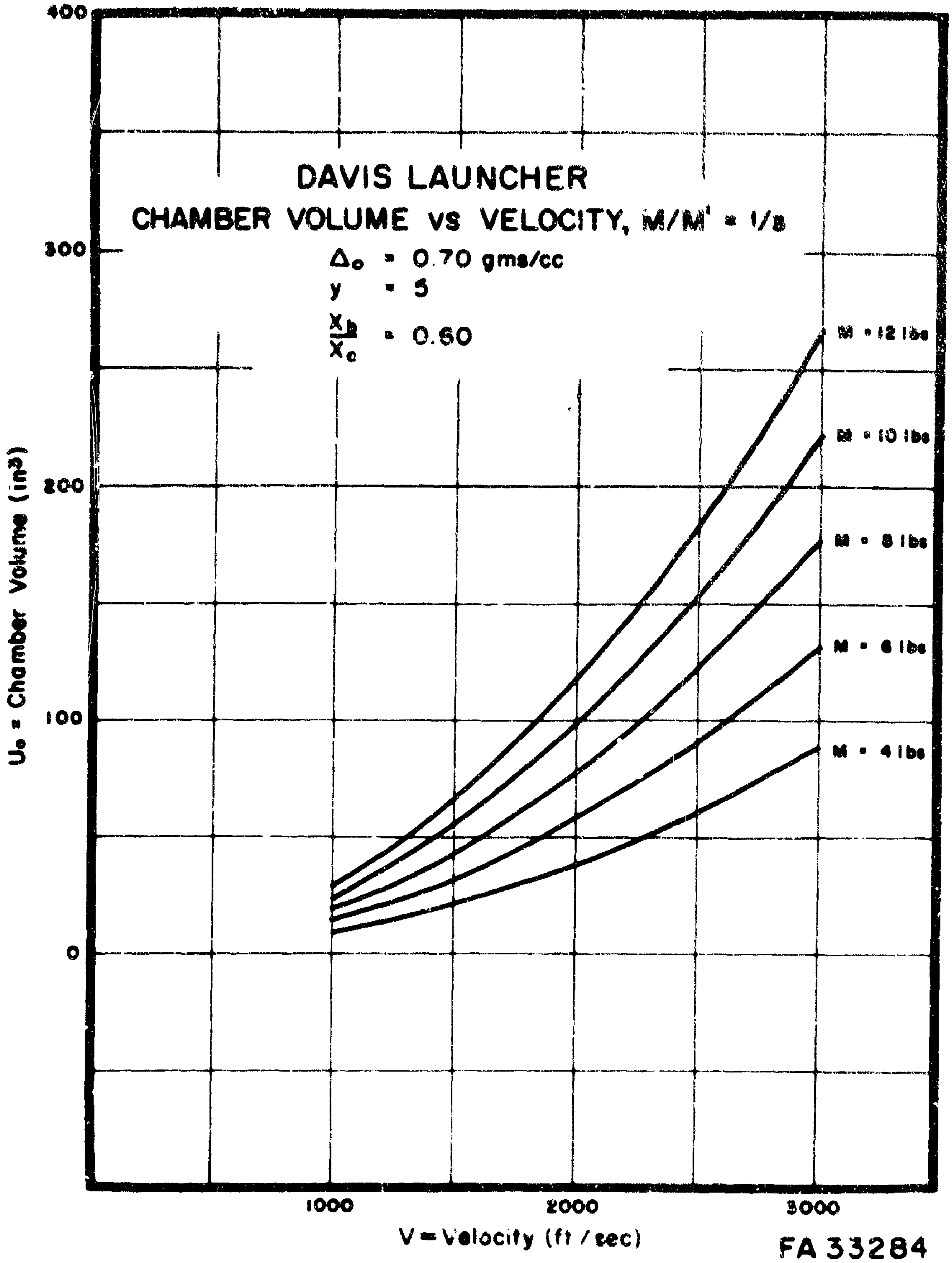

Figure 7. Chamber Volume vs Velocity M/M' = 1/3

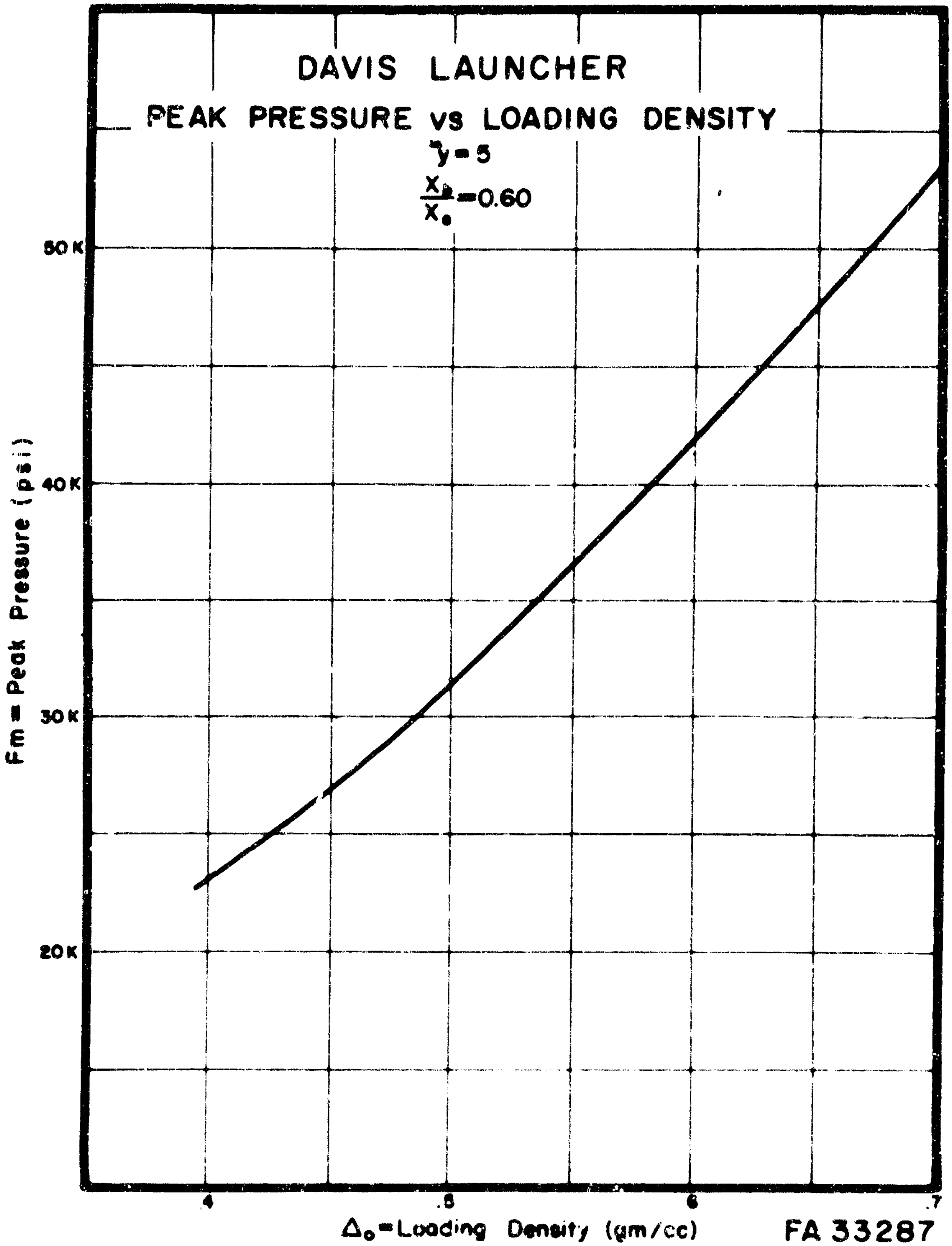

Figure 8. Peak Pressure vs Loading Density

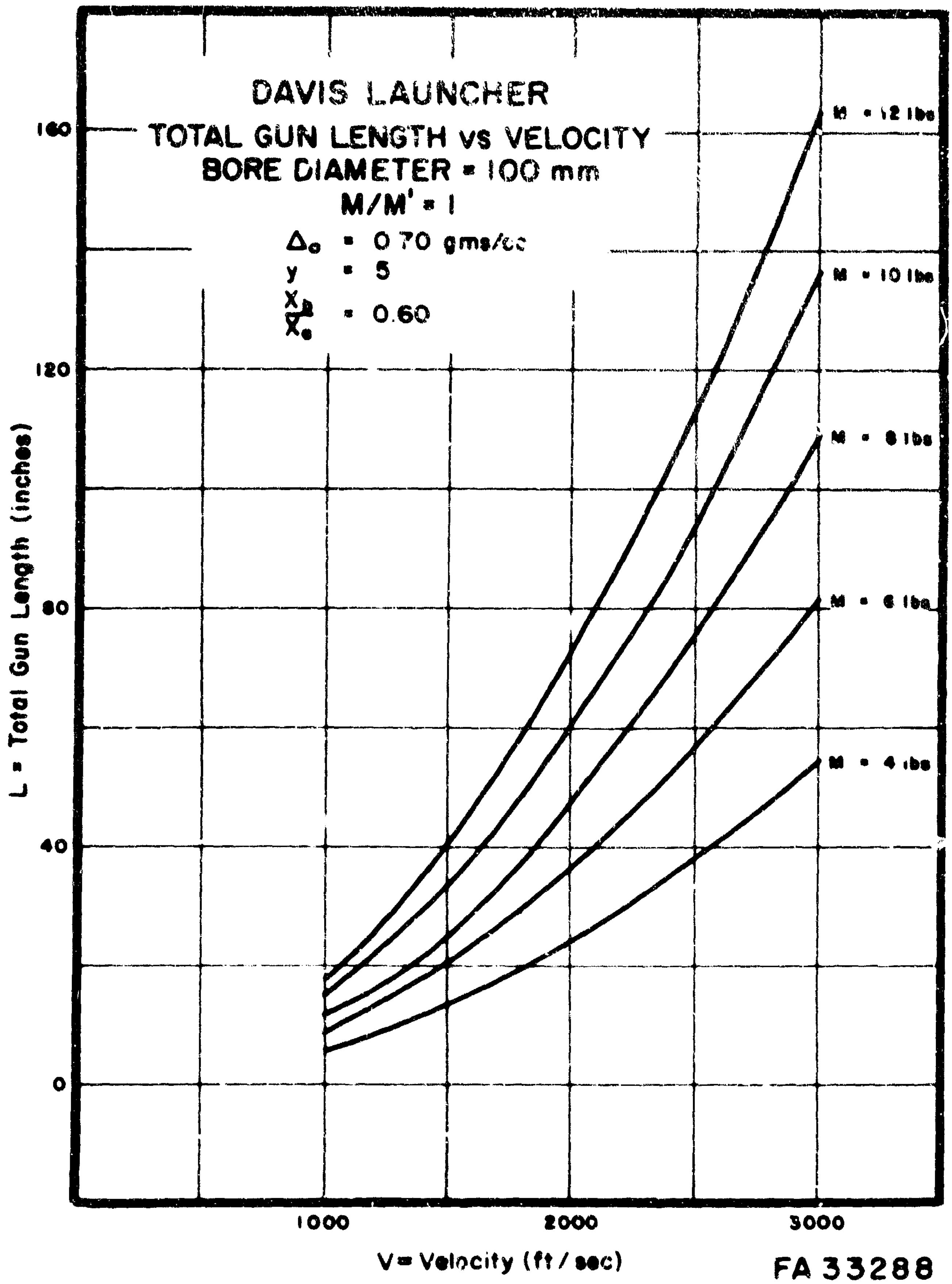

Figure 9. Total Gun Length vs Velocity—Bore Diameter = 100 mm M/M' = 1

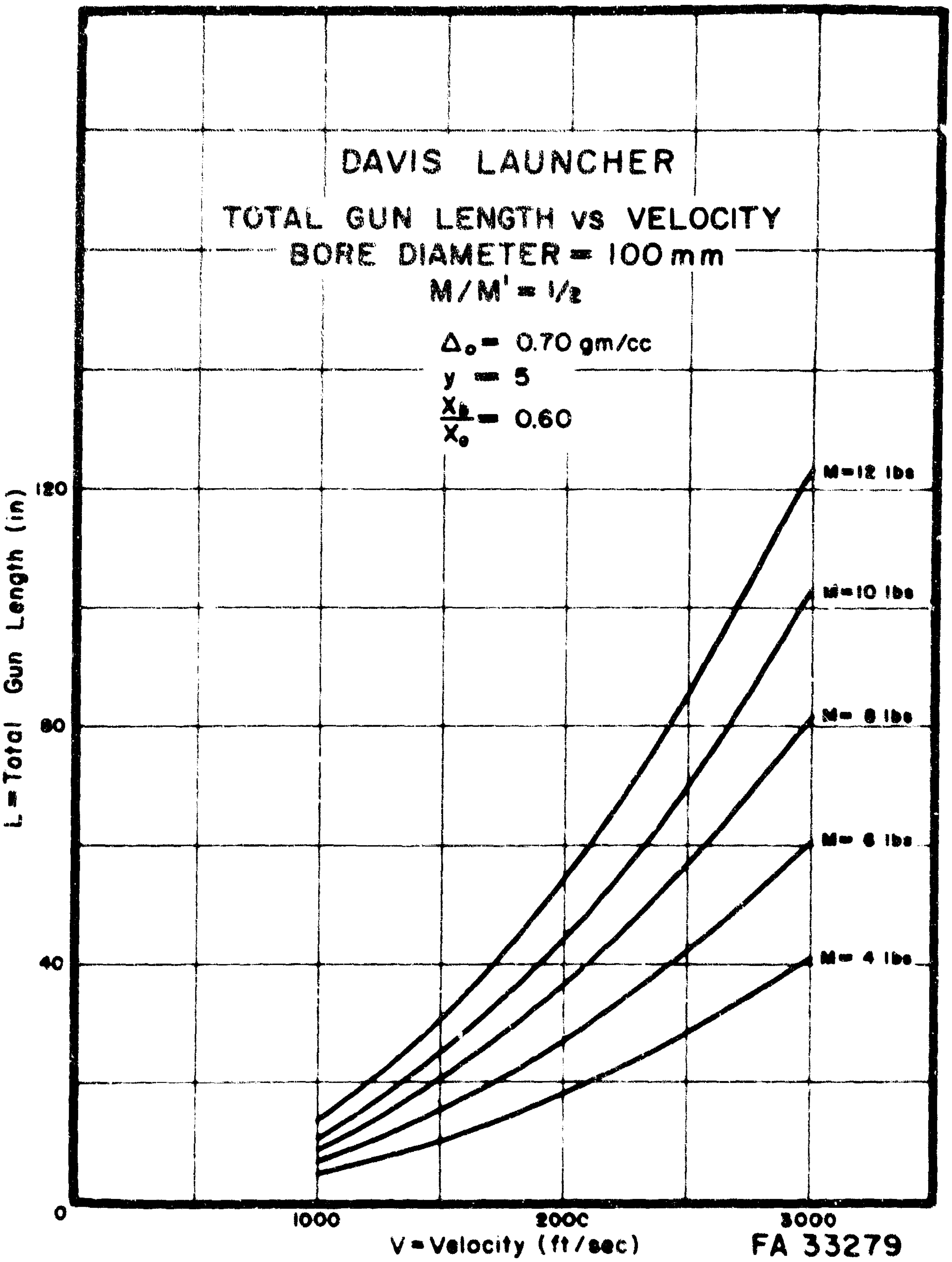

Figure 10. Total Gun Length vs Velocity—Bore Diameter = 100 mm M/M' = 1/2

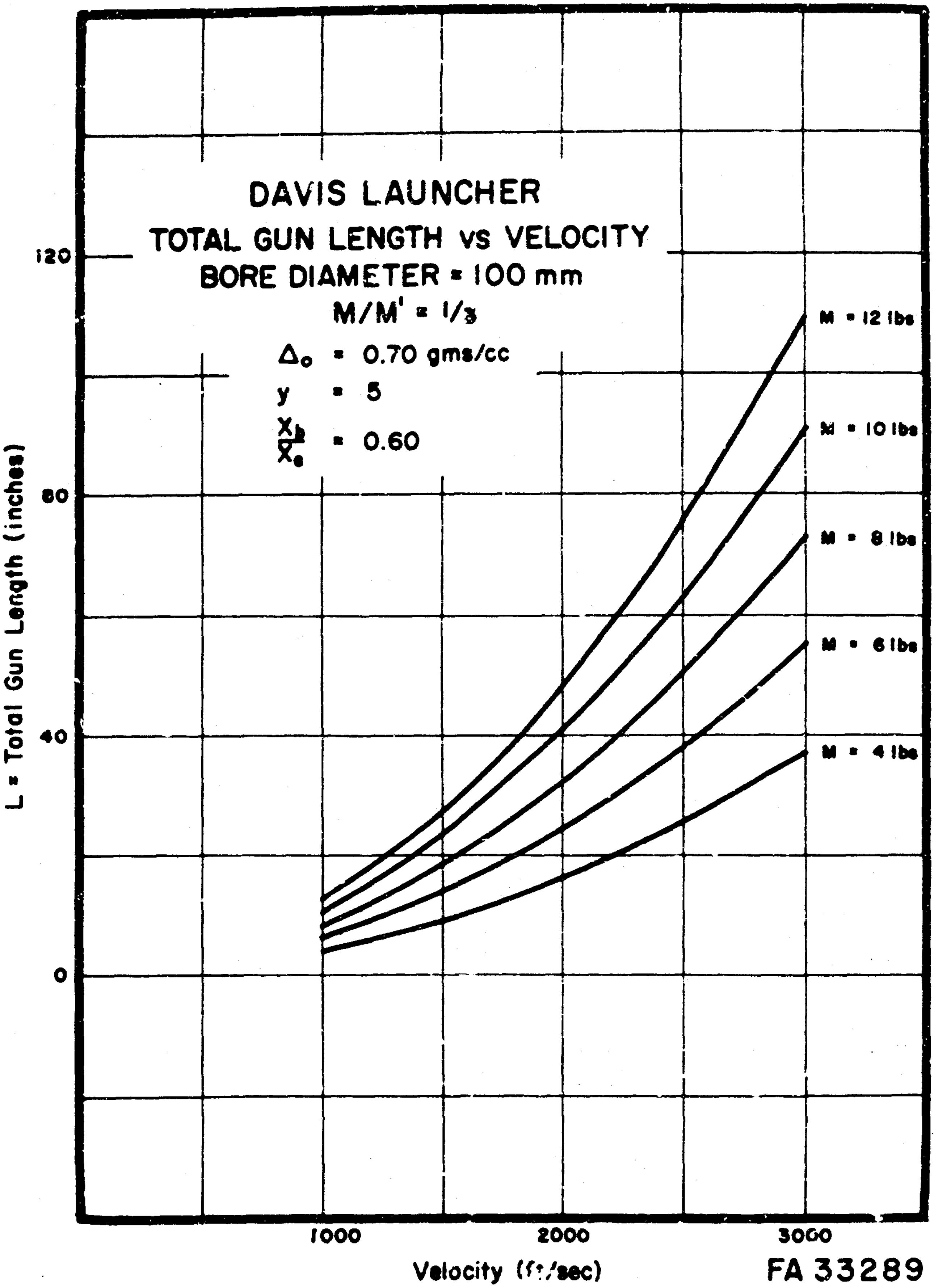

Figure 11. Total Gun Length vs Velocity. Bore Diameter = 100 mm
M/M' = 1/3

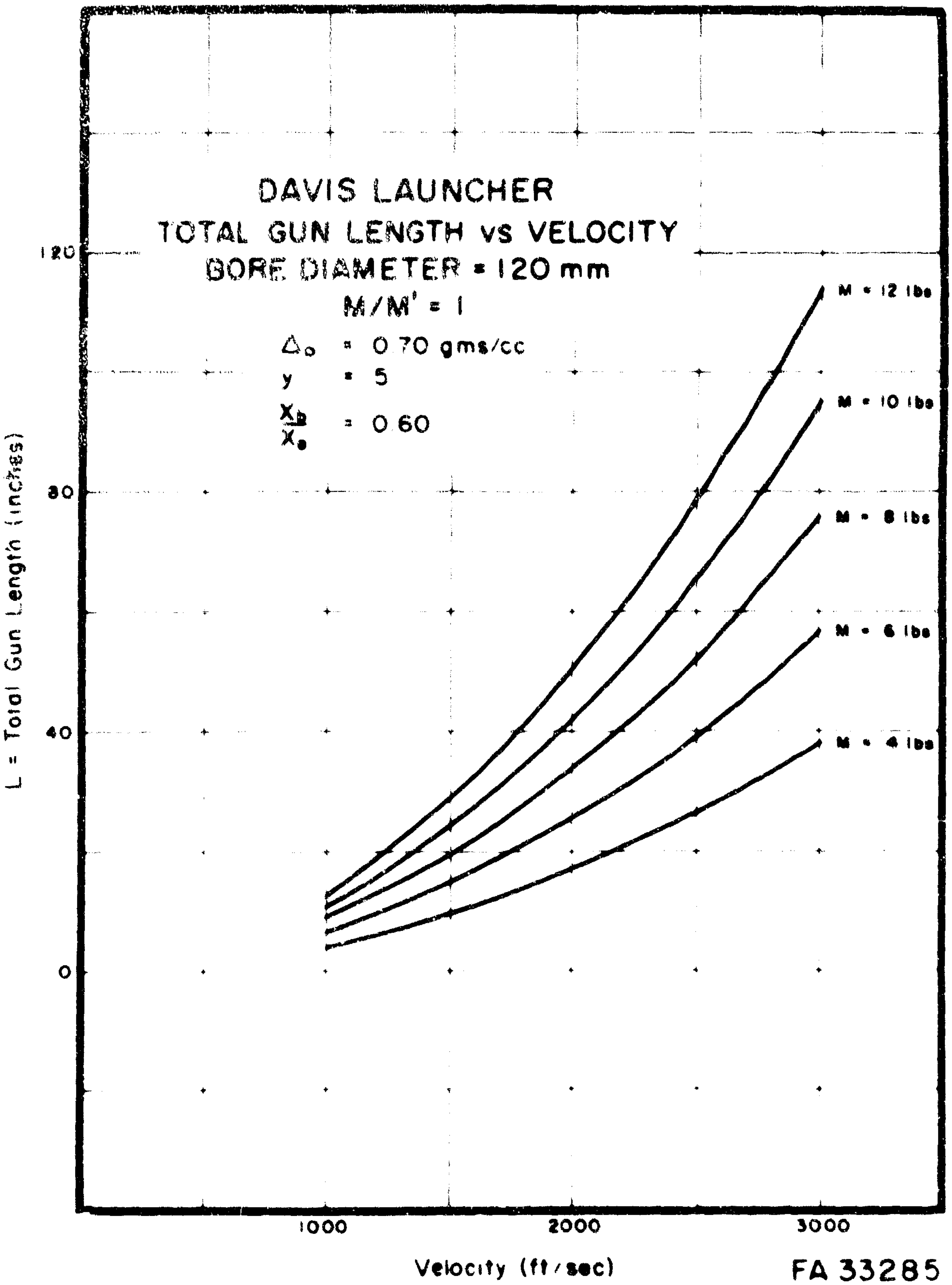

Figure 12. Total Gun Length vs Velocity - Bore Diameter = 120 mm
M/M' = 1

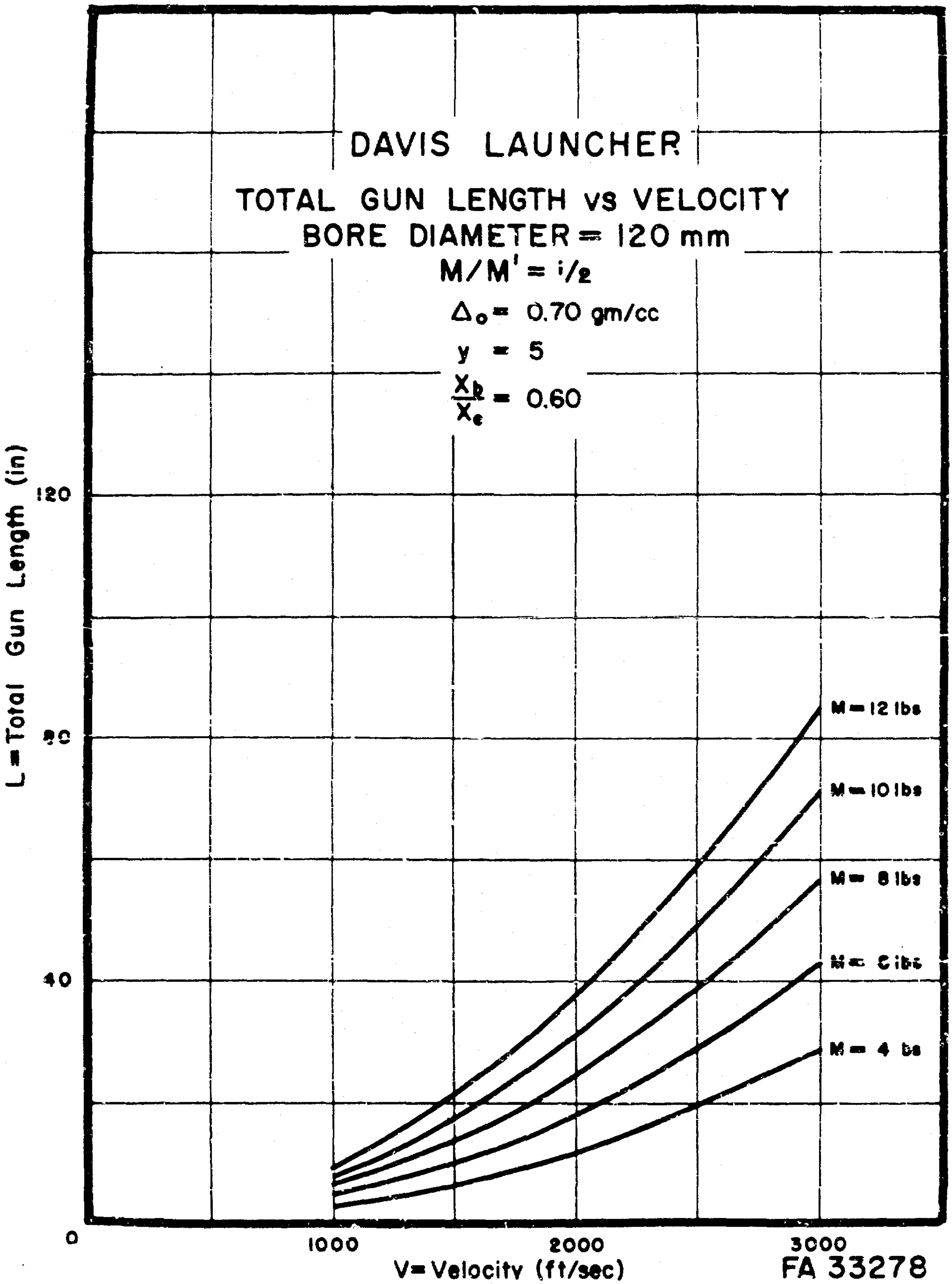

Figure 13. Total Gun Length vs Velocity - Bore Diameter = 120 mm M/M' = 1/2

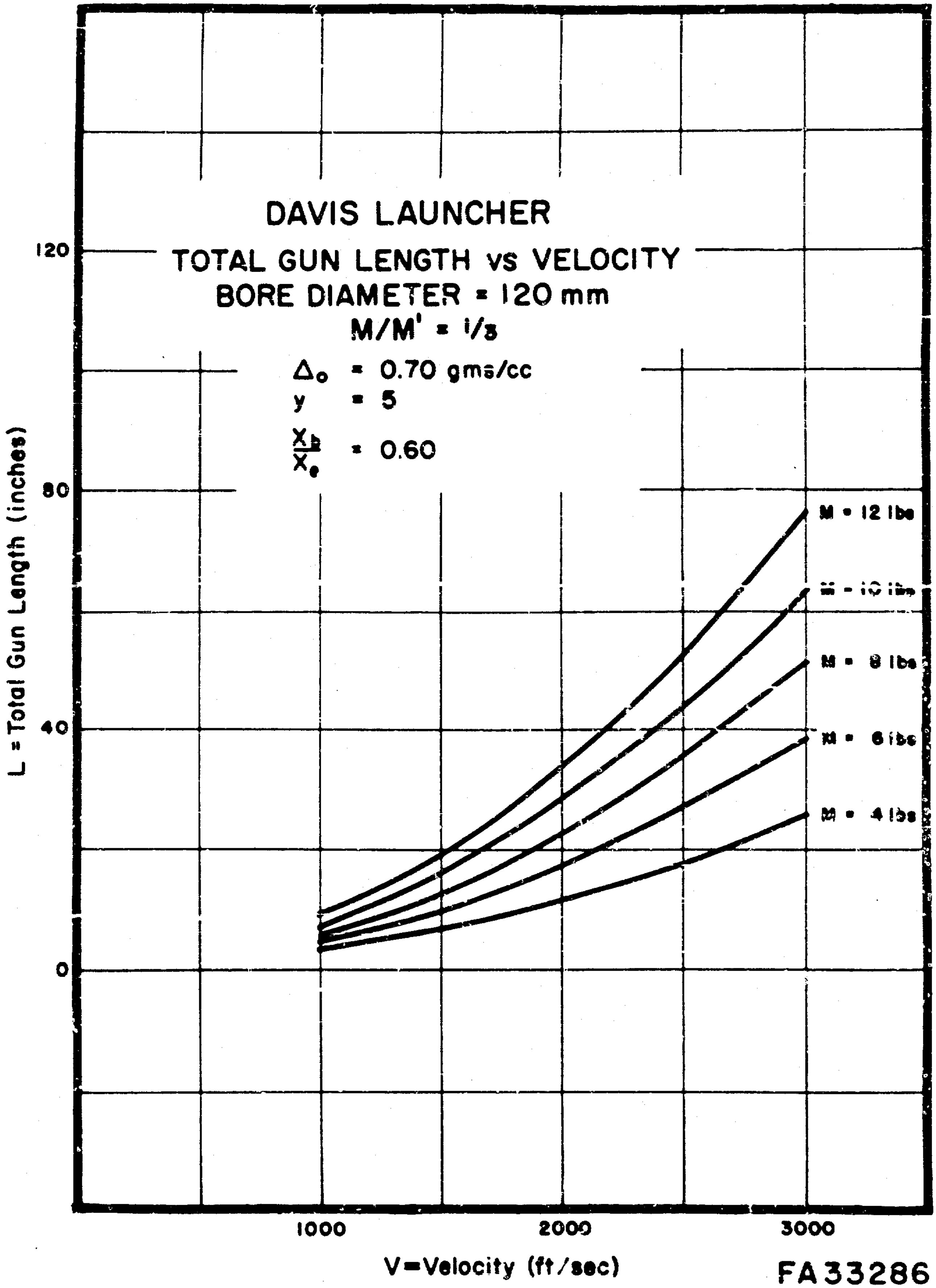

Figure 14. Total Gun Length vs Velocity - Bore Diameter = 120 mm $M/M' = 1/3$

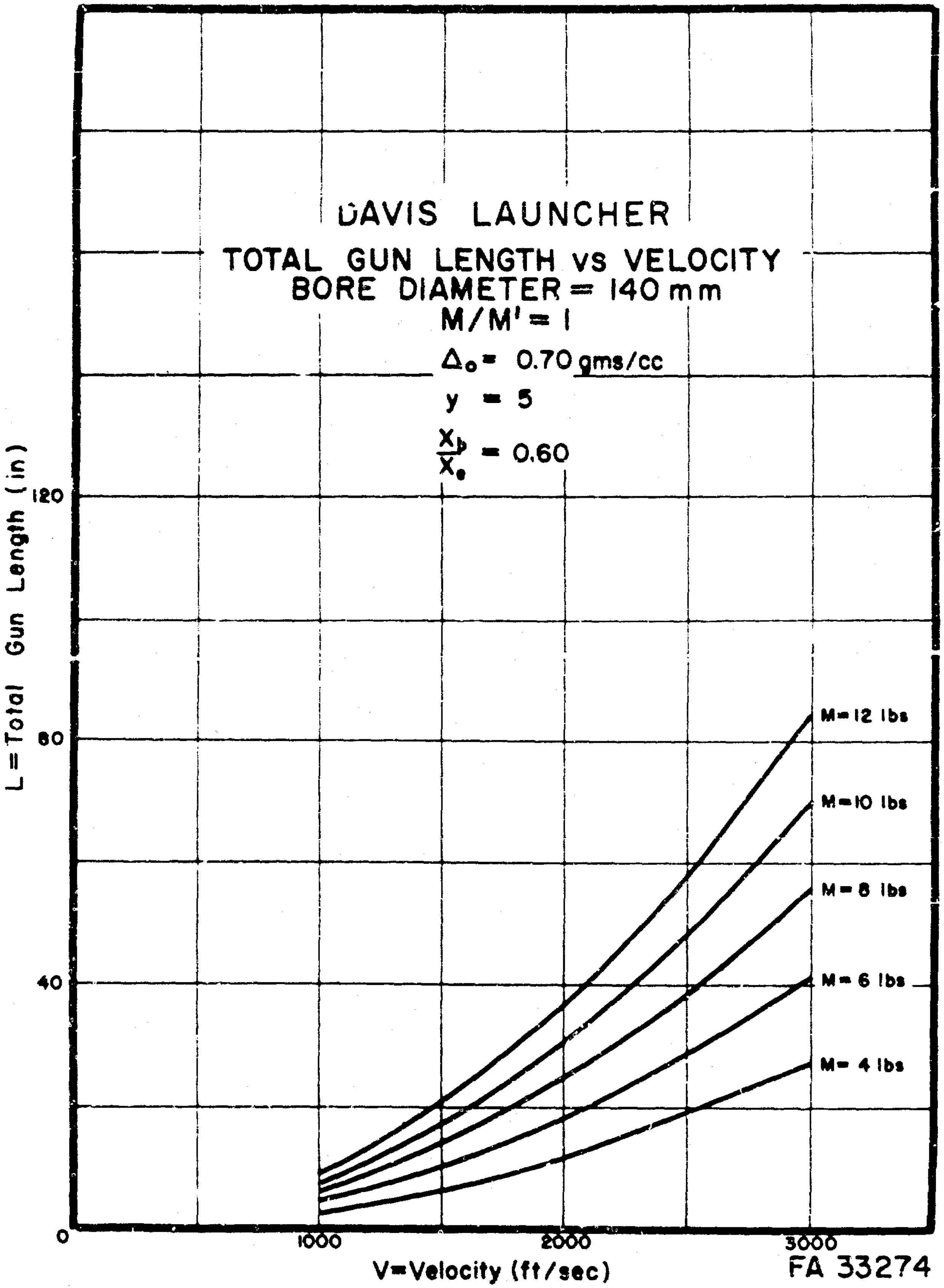

Figure 15. Total Gun Length vs Velocity - Bore Diameter = 140 mm M/M' = 1

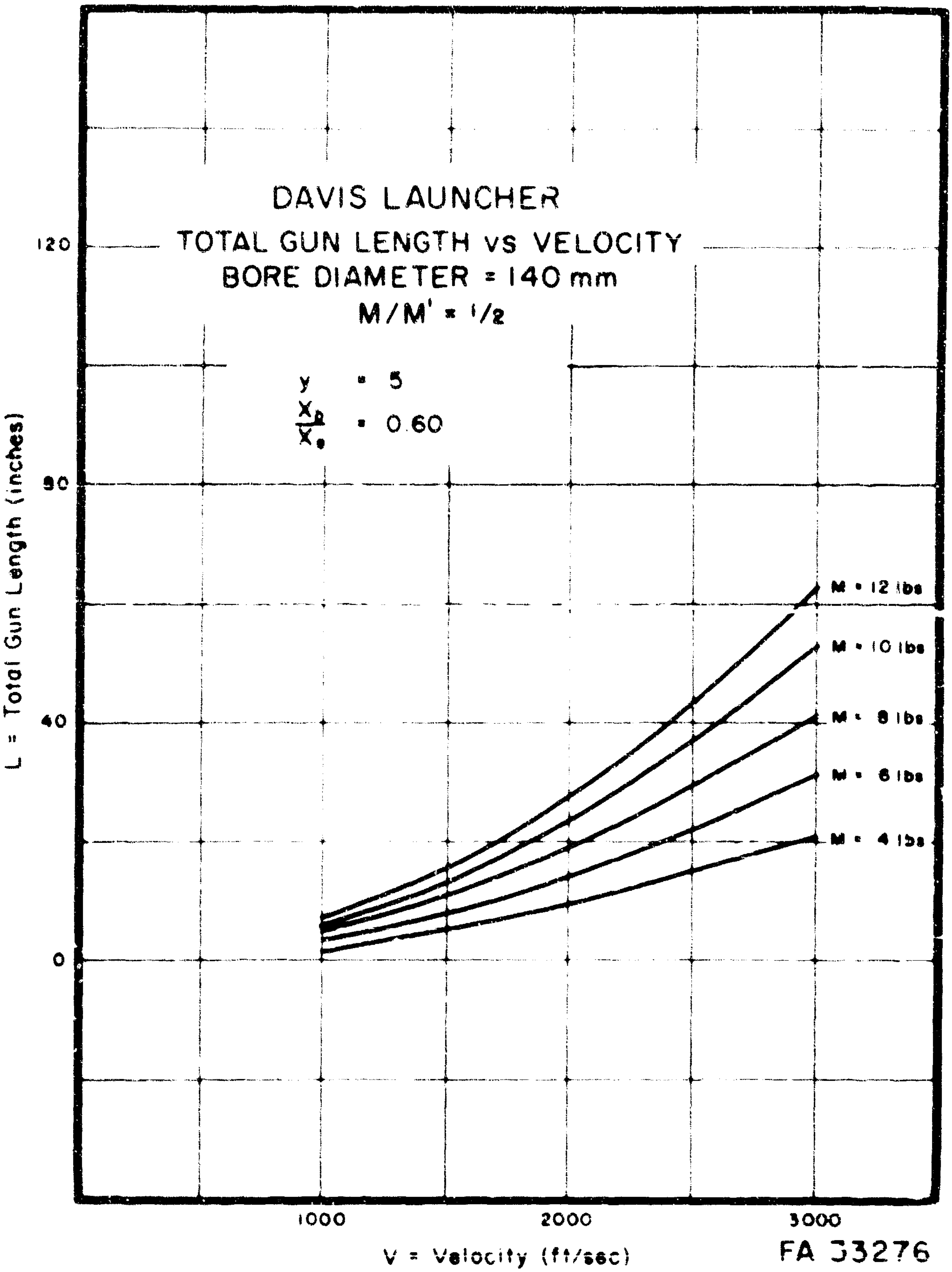

Figure 16. Total Gun Length vs Velocity - Bore Diameter = 140 mm
M/M' = 1/2

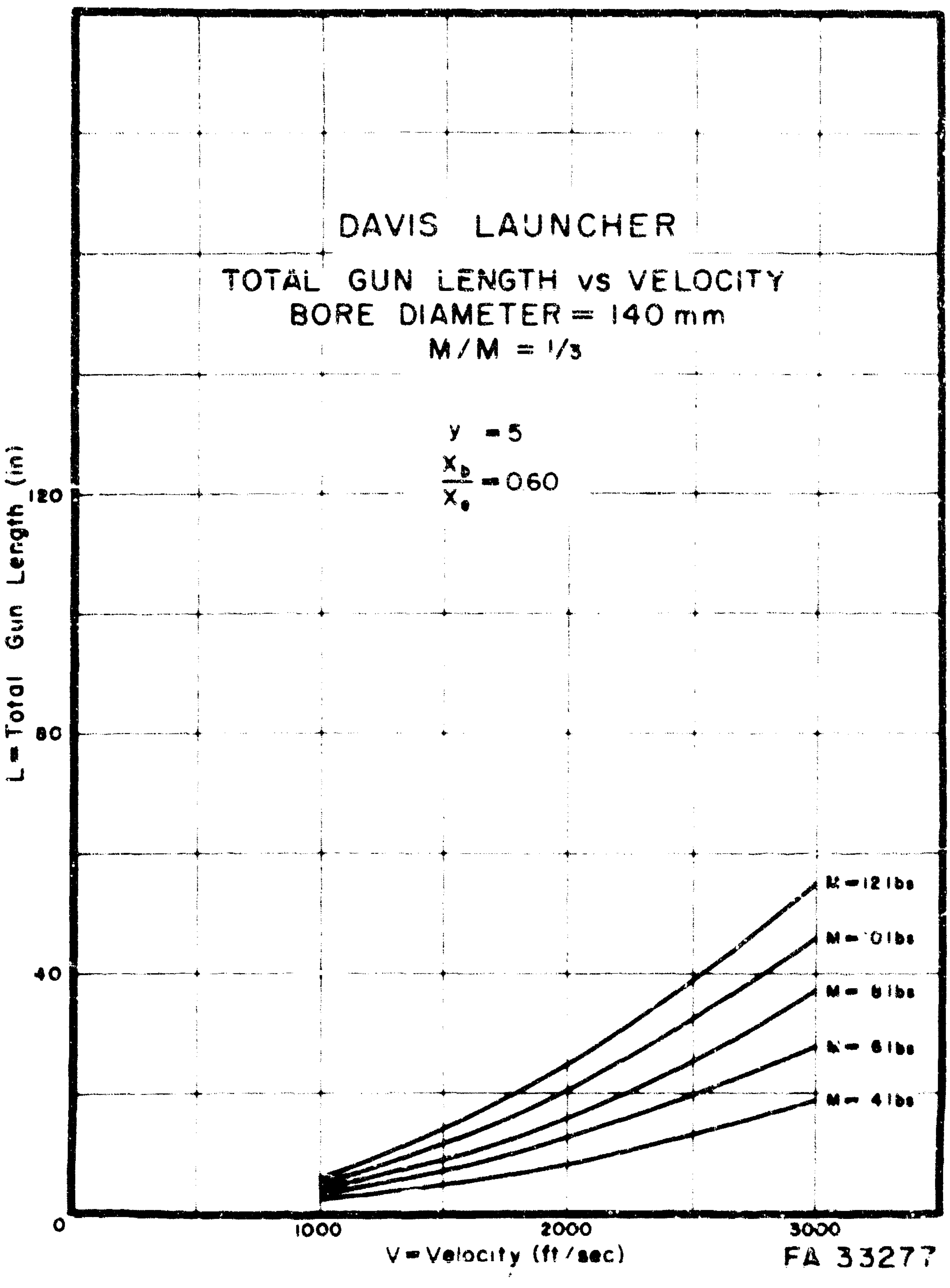

Figure 17. Total Gun Length vs Velocity - Bore Diameter = 140 mm M/M' = 1/3

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13. ABSTRACT

A set of interior ballistic equations is derived for a reactionless type of launcher of the Davis Gun type in which two masses are ejected from a common chamber.

Based on ballistic analysis of the Davis Gun launcher it appears feasible that a high performance type Davis Gun launcher could be effectively used from a vehicle.